Note Title 5/2/2019

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} = x[0] + x[1] z^{-1} + x[2] z^{-2} + x[3] z^{-3} + \cdots$$

Ex

$$X(z) = \frac{z}{z-1}$$

Long Division:

$$\begin{array}{r} 1 + z^{-1} + z^{-2} + z^{-3} + \cdots \\ z-1 \;\overline{)\; z} \\ \underline{z - 1} \\ 1 \\ \underline{1 - z^{-1}} \\ z^{-1} \\ \underline{z^{-1} - z^{-2}} \\ z^{-2} \\ \underline{z^{-2} - z^{-3}} \\ z^{-3} \\ \vdots \end{array}$$

$\therefore x[0] = 1;\ x[1] = 1,\ x[2] = 1$

$x[3] = 1;$ ⋯

$\therefore \boxed{x[n] = u[n]}$

Ex $\quad X(z) = \dfrac{z^2 - 1}{z^3 + 2z + 4}$

$$\begin{array}{r} z^{-1} - 3z^{-3} - 4z^{-4} + 6z^{-5} + \cdots \\ z^3 + 2z + 4 \;\overline{)\; z^2 - 1} \\ \underline{z^2 + 2 + 4z^{-1}} \\ -3 - 4z^{-1} \\ \underline{-3 - 6z^{-2} - 12z^{-3}} \\ -4z^{-1} + 6z^{-2} + 12z^{-3} \\ \underline{-4z^{-1} - 8z^{-3} - 16z^{-4}} \\ 6z^{-2} + 20z^{-3} + 16z^{-4} \\ \vdots \end{array}$$

$$\longrightarrow X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} = x[0] + x[1] z^{-1} + x[2] z^{-2} + x[3] z^{-3} + \cdots$$

$x[0] = 0;\ x[1] = 1;\ x[2] = 0;\ x[3] = -3$

$x[4] = -4;\ x[5] = 6$

$$e^{-t} \leftrightarrow \frac{1}{s+1}$$

$$\frac{a}{s+b} \leftrightarrow a e^{-bt}$$

$$a^n u[n] \leftrightarrow \frac{z}{z-a}$$

$$(-a)^n u[n] \leftrightarrow \frac{z}{z+a}$$

Ex $\quad X(z) = \dfrac{z}{z^2-1}$

$$\frac{X(z)}{z} = \frac{1}{z^2-1} = \frac{1}{(z+1)(z-1)} = \frac{A}{(z+1)} + \frac{B}{(z-1)}$$

$$A = (z+1)\left(\frac{X(z)}{z}\right)\Big|_{z=-1} = (z+1)\left(\frac{1}{(z+1)(z-1)}\right)\Big|_{z=-1} = \frac{1}{-1-1} = -\frac{1}{2}$$

$$B = (z-1)\left(\frac{X(z)}{z}\right)\Big|_{z=1} = (z-1)\left(\frac{1}{(z+1)(z-1)}\right)\Big|_{z=1} = \frac{1}{1+1} = \frac{1}{2}$$

$$\frac{X(z)}{z} = \frac{-1/2}{z+1} + \frac{1/2}{z-1}$$

$$\therefore X(z) = -\frac{1}{2}\left(\frac{z}{z+1}\right) + \frac{1}{2}\left(\frac{z}{z-1}\right)$$

$$\therefore x[n] = -\frac{1}{2}(-1)^n u[n] + \frac{1}{2}(1)^n u[n]$$

Q2 $x[n] = \frac{1}{2} - \frac{1}{2}(-1)^n$, $n = 0, 1, 2, \cdots$

Ex $X(z) = \frac{z^2+1}{z^2-1} \Rightarrow \frac{X(z)}{z} = \frac{z^2+1}{z(z^2-1)} = \frac{z^2+1}{z(z+1)(z-1)}$

$$= \frac{A}{z} + \frac{B}{z+1} + \frac{C}{z-1}$$

$$A = z\left(\frac{X(z)}{z}\right)\Big|_{z=0} = z\left(\frac{z^2+1}{z(z+1)(z-1)}\right)\Big|_{z=0} = \frac{1}{(1)(-1)} = -1$$

$$B = (z+1)\left(\frac{X(z)}{z}\right)\Big|_{z=-1} = (z+1)\left(\frac{z^2+1}{z(z+1)(z-1)}\right)\Big|_{z=-1} = \frac{1+1}{(-1)(-1-1)} = \frac{2}{2} = 1$$

$$C = (z-1)\left(\frac{X(z)}{z}\right)\Big|_{z=1} = (z-1)\left(\frac{z^2+1}{z(z+1)(z-1)}\right)_{z=1} = \frac{1+1}{(1)(1+1)} = 1$$

$$\frac{X(z)}{z} = \frac{-1}{z} + \frac{1}{z+1} + \frac{1}{z-1} \Rightarrow X(z) = -1 + \frac{z}{z+1} + \frac{z}{z-1}$$

$$x[n] = -\delta[n] + (-1)^n u[n] + u[n]$$

Ex $X(z) = \frac{6z^3 + 2z^2 - z}{z^3 - z^2 - z + 1} = \frac{6z^3 + 2z^2 - z}{(z-1)^2(z+1)}$

$$\frac{X(z)}{z} = \frac{6z^2 + 2z - 1}{(z-1)^2(z+1)} = \frac{A}{(z-1)} + \frac{B}{(z-1)^2} + \frac{C}{z+1}$$

$$C = (z+1)\left(\frac{X(z)}{z}\right)\Big|_{z=-1} = 0.75$$

$$B = (z-1)^2 \left(\frac{X(z)}{z}\right)\Big|_{z=1} = 3.5$$

$$A = \frac{d}{dz}(z-1)^2 \frac{X(z)}{z}\Big|_{z=1} = \frac{d}{dz}\left(\frac{6z^2+2z-1}{z+1}\right)\Big|_{z=1} = 5.25$$

$$X(z) = \frac{5.25z}{z-1} + \frac{3.5z}{(z-1)^2} + \frac{.75z}{z+1}$$

$$x[n] = 5.25\,u[n] + 3.5n\,u[n] + 0.75(-1)^n u[n]$$

$y[n] + ay[n-1] = bx[n]$; find the Transfer function $H(z)$

$$Y(z) + a z^{-1} Y(z) = b X(z) \qquad * z$$

$$zY(z) + aY(z) = bzX(z)$$

$$Y(z)[z+a] = bzX(z) \Rightarrow H(z) = \frac{bz}{z+a}$$

Ex

$$y[n+2] + 3y[n] + 2y[n-1] = 4x[n-1]$$

Find $H(z)$

$$z^2 Y(z) + 3Y(z) + 2z^{-1}Y(z) = 4z^{-1}X(z) \qquad \times z$$

$$z^3 Y(z) + 3zY(z) + 2Y(z) = 4X(z)$$

$$\therefore H(z) = \frac{4}{z^3+3z+2}$$

Ex

$$y[n] + 1.5\,y[n-1] + 0.5\,y[n-2] = x[n] - x[n-1]$$

$$Y(z) + 1.5\,z^{-1}Y(z) + 0.5\,z^{-2}Y(z) = X(z) - z^{-1}X(z) \qquad * z^2$$

$$z^2 Y(z) + 1.5\,z\,Y(z) + 0.5\,Y(z) = z^2 X(z) - z\,X(z)$$

$$H(z) = \frac{z^2 - z}{z^2 + 1.5z + 0.5}$$